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Modeling of non-isothermal gas flow through a heterogeneous medium

A. BUIKIS and N. ULANOVA
Institute of Mathematics, Latvian Academy of Science and University of Latvia, Riga, Latvia

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Abstract—A porous media model as a system of two interacting imaginary media is proposed to solve the problem of studying the process of heating of solar thermal station reactor, which is a large long tube filled with rock. The heating is the result of a motioned heat gas flow with the initial constant rate and temperature. The process is described by two one-dimensional non-stationary differential equations of heat transfer convection and complemented by the equation of mass conservation and temperature dependence for gas density. For the difference realization of obtained problems with the solution, which has a large gradient in a restricted area, a differential approximation method with artificial viscosity coefficient introduction is used. The behavior of a difference solution in the vicinity of area with abrupt temperature change, depending on introduced pseudo viscosity, is studied. The specific applied problem simplified model with the constant coefficient equation, which allows essentially reduced computing time, has been used.

INTRODUCTION

To coordinate the requirements of power engineering and ecology, the problems of study and application of non-traditional energy sources are at present of particular interest. These include also direct use of solar energy. A part of solar power plants accumulating heat is the reactor. For example, it is a large 25 m long tube with a diameter of 14 m, which is filled with material having good capacity characteristics (granite rock) [1]. The reactor is operated in sequential removable regimes. The first is the charging cycle, when motioned flow with a constant rate heats the rock when passing through it. The second—the discharging cycle—is a reverse process, when cold air, when passing in reverse direction through the rock is heated by it up to an operating temperature.

A similar situation, which is intended to have maximum use of heat and its storage in large volumes, is common in many fields of research and technology. Various methods were employed in the mathematical modeling of these processes. We present a brief review of the literature on the solution of the above or similar problems [1–6]. We will first consider a specific applied solution of the problem and will formulate the class of analogous problems. Then we will describe and justify the method chosen by us.

REVIEW OF LITERATURE

A model of imaginary continuous medium often serves as an approximation in solving the problems of heterogeneous media. In the case of the above problem, in refs. [2–5] the consideration of the total mass of rock loaded in the reactor as a certain unit body divided into sections along the direction of the gas flows and having a constant temperature has been proposed. In other works, for example refs. [1, 6], rock is shown as a discrete medium (a sphere with an equivalent diameter D_k) divided into sphere layers with a constant temperature. In each layer the equation of the Fourier heat conductivity is considered for one sphere, assuming that the conditions are the same for each sphere in the fixed layer along the direction of gas flow. The particles of rock and gas have different temperatures, as they differ also in the various layers. Moreover, the dependence of flow thermal physical parameters on temperature is also taken into account in the coefficients of equations, i.e. the problem is nonlinear. In the case of the model described in refs. [1, 6] the important role is played by the heat transfer coefficient α, between each spherical particle of material and moving gas flow. The above coefficient in ref. [1] is found by taking into account the characteristics of the flow of gas and solid particles of the rock

$$\alpha = \frac{\alpha_0}{1+\alpha_0 F_k/(2.V_m c_f)}, \tag{1}$$

here

$$\alpha_0 = 0.33\lambda_f Re^{0.6}/2R_k,$$

$$V_m = \rho_f v_s V_1/h_s.$$

Such formulation allows one to study this problem in sufficient detail. However, since separate equations are considered for each layer, this leads to the necessity of solving large stiff systems of equations which

NOMENCLATURE

c	heat capacity
F_k	sphere surface
h	spatial step
h_s	height of one sphere layer
m	porosity
R_k	radius of the sphere
T	temperature
T_0	initial temperature of the two media
T_1	temperature of air blown
$\mathbf{v}$	air velocity
v_0	initial velocity of air stream
v_s	mean flow velocity in one sphere layer
V_1	air volume around each sphere.

Greek symbols

α	inter-component heat transfer coefficient
λ	air heat conductivity
ρ	density
τ	time step.

Subscripts

f	gas
k	rock.

require lengthy computing time. In an attempt to take into account the temperature change of material along the radius of the reactor (in each layer), such formulation of the problem becomes bulky.

DIFFERENTIAL FORMULATION OF PROBLEM

In principle, we shall deal with other formulations of the problem for the solution of a similar problem with an account of the above technical difficulties. If one assumes that the porous body is two interacting imaginary media [7], modeling both the solid and gaseous (liquid) phases, the heat transfer process could be presented as a system of two equations

$$\begin{cases} mc_f\rho_f\dfrac{\partial T_f}{\partial t} = -c_f\rho_f\mathbf{v}\,\overrightarrow{\text{grad}}\,T_f \\ \quad +\text{div}(\lambda_f\overrightarrow{\text{grad}}\,T_k)+\alpha(T_k-T_f) \\ (1-m)c_k\rho_k\dfrac{\partial T_k}{\partial t} = \text{div}(\lambda_k\overrightarrow{\text{grad}}\,T_k)+\alpha(T_f-T_k). \end{cases} \tag{2}$$

The seeming simplicity of the above system and each of its parameters, λ and α in particular, involves complex dependencies taking into account the properties of all media. If the flow velocity is sufficiently large, heat transfer caused by convection considerably exceeds the heat transfer due to heat conductivity, which allows one to neglect the heat conductivity terms of equations (2). To investigate the polyphase mediums, such an approach is used in the description of fraction micromotion [8].

The analogous equation systems are also obtained by Shcherban *et al.* [9], when studying the processes of the one-dimensional filtration of heat carrier on a fractured medium under its thermal non-uniform conditions. When taking into account the dependence of heat carrier density on temperature, the equation of mass conservation and the dependence of density on temperature are added to the system of equations

$$\begin{cases} -\dfrac{\partial\rho_f}{\partial t} = \dfrac{\partial}{\partial x}(\rho_f v) \\ \rho_k c_k(1-m)\dfrac{\partial T_k}{\partial t} = -\alpha(T_k-T_f) \\ c_f\rho_f m\dfrac{\partial T_f}{\partial t} = -c_f\dfrac{\partial}{\partial x}(v\rho_f T_f)+\alpha(T_k-T_f) \\ \rho_f = f(T_f). \end{cases} \tag{3}$$

The authors of ref. [9] demonstrated that this interpretation of the problem permits one to describe the given process of filtration with sufficient accuracy.

The formulation of problems for finding temperature fields in oil beds, with an account of temperature difference of solid bed and filtrating liquid, may serve as other examples. The first similar formulations for thermally insulated beds were considered in ref. [10], while the problem with an account of heat transfer with the surrounding substances is discussed in refs. [11–13].

Based on the above models for similar problems, we consider a suitable formulation, which is described by using a porous medium model as the system of interacting imaginary media.

The 'gas–solid body' system is represented as two one-dimensional transient differential equations. In the equation of the substance, only heating of separate substance particles in each point of one-dimensional space due to the convective heat transfer of gas is considered, whereas heat conductivity and contact heat transfer between the particles are neglected. The system is supplemented by the equation of mass conservation and temperature dependence for gas density, which allows one to take into account the change of flow velocity along the direction of motion

$$\begin{cases} m\dfrac{\partial}{\partial t}\left[\rho_f\left(c_fT_f+\dfrac{v^2}{2}\right)\right]+\dfrac{\partial}{\partial x} \\ \quad\times\left\{v\left[\rho_f\left(c_fT_f+\dfrac{v^2}{2}\right)\right]\right\}=\alpha(T_k-T_f) \\ (1-m)\rho_kc_k\dfrac{\partial T_k}{\partial t}=\alpha(T_f-T_k) \\ -m\dfrac{\partial\rho_f}{\partial t}=\dfrac{\partial(\rho_fv)}{\partial x} \\ \rho_f=\rho_0/T_f. \end{cases} \tag{4}$$

The initial and boundary conditions may be written as

$$T_f|_{x=0}=T_1 \quad (\rho_fv)|_{x=0}=\rho_0v_0$$
$$T_f|_{t=0}=T_0 \quad T_k|_{t=0}=T_0, \tag{5}$$

here $\rho_0=\rho_f(T_0)$.

To solve this system, a new variable—the mass velocity $G=\rho_fv$—is introduced and system (4) could be written as

$$\begin{cases} \dfrac{mc_f}{v}\dfrac{\partial(GT_f)}{\partial t}+c_f\dfrac{\partial(GT_f)}{\partial x} \\ \quad+\dfrac{1}{2}\left[\dfrac{v\,\partial G}{\partial t}+v^2\dfrac{\partial G}{\partial x}\right]=\alpha(T_k-T_f) \\ (1-m)\rho_kc_k\dfrac{\partial T_k}{\partial t}=\alpha(T_f-T_k) \\ -\dfrac{m}{v}\dfrac{\partial G}{\partial t}=\dfrac{\partial G}{\partial x} \\ \rho_f=\rho_0/T_f. \end{cases} \tag{4'}$$

It is clear that the first equation of the above system (4) gives form to the convective heat front in the air component, whereas the inter-component heat transfer coefficient α becomes the value determining the temperature distribution of both air and rocks. Taking into account how important this parameter is, various methods of its determination [5, 13, 15] were considered. We used a version, proposed in ref. [15], where the solution of non-stationary equation of heat conductivity for a sphere was employed to obtain an expression for finding the value of the volume coefficient of heat transfer. The average value of heat transfer was found from the conditions of thermal balance, which could be written as

$$\alpha=15\lambda_k/R_k^2.$$

R_k is the radius of the sphere, i.e. the equivalent radius of rock stones.

In order to compare the proposed formulation of the problem with the one described in ref. [5], the remaining initial data relevant to an analogous process should be employed. Then, at $R_k=0.0125$ m, $\alpha=123000$ W m^{-2} grad. At such large values of α, the temperature profile obtained as a result of the solution of the system of equations given in equations (4) and (5) with jump conditions at the tube entrance in an initial moment of time, would approach that of a rectangle, i.e. a practically discontinuous solution is obtained.

STUDY OF DIFFERENCE PROBLEM

A difference realization of the problem, in which the width of temperature front is estimated by several steps within space, might generate the oscillating solutions caused by dispersion errors of difference scheme. Besides, the schemes of the first order of accuracy are not sufficiently precise for calculating such problems. Hence, the difference realization of the system (4). Equation (5) is of particular attention. Since the method of solution of the second equation of equation (4) does not cause any difficulties, its choice in the range from exact analytical solutions to various discrete schemes was based on such criterion as speed of computation. We chose an implicit difference scheme 'with weight' Krank–Nicolson type

$$\frac{T_{(k)j}^{n+1}-T_{(k)j}^{n}}{\tau}=\frac{\alpha}{(1-m)c_k\rho_k}\times\tfrac{1}{2}[(T_{(f)j}^{n+1}-T_{(k)j}^{n+1})+(T_{(f)j}^{n}+T_{(k)j}^{n})].$$

In order to solve the first equation of (4) we will use an approach based on the introduction of artificial viscosity (pseudo viscosity), which allows us to describe approximately almost the discontinuous solution. The behavior of the discrete solution in the vicinity of discontinuity depends essentially on the type of introduced pseudo viscosity, as well as on the scheme viscosity caused by the approximation error of the initial equation. Their effect can be evaluated by means of the differential approximation method [16–18] estimating the value of introduced viscosity. Differential approximation of difference scheme is an differential equation obtained from the difference scheme approximated by it. Hence, it is different for various difference approximations of the same equation. In the case of the first equation (4) (with constant coefficients), where $u=v_0/m$, the first differential approximation can be written in the form

$$\rho_fc_f\frac{\partial T_f}{\partial t}+u\rho_fc_f\frac{\partial T_f}{\partial x}-a\frac{\partial^2T_f}{\partial x^2}=\alpha(T_k-T_f),$$

where a is the value of introduced pseudo viscosity assuming the following values [19]:

when using the scheme with one-side differences

$$a=\frac{uh}{2}\left(1-\frac{\tau u}{h}\right),$$

when using the scheme of the second order of precision (Lacks–Vendrof type)

$$a = \frac{h^2}{2\tau}\left(1 - \frac{\tau^2 u^2}{h^2}\right).$$

Here the result of using notion of the first differential approximation of difference scheme is substantiation of fact that introduction of diffusion terms with artificial viscosity coefficient does not change the physical formulation of the problem equations (4) and (5), but serves as a means of realization of the first equation from (4′).

Finally, the difference scheme for the first and third equation of (4) is written as

$$\frac{G_j^{n+1}T_j^{n+1} - G_j^n T_j^n}{\tau} + \frac{V_j^n}{2m}\left[G_j^n \frac{T_{j+1}^n - T_{j-1}^n}{2h} + G_j^{n+1}\frac{T_{j+1}^{n+1} - T_{j-1}^{n+1}}{2h}\right] + \frac{(V_j^n)^2}{2mc_f}\left[\frac{G_j^{n+1} - G_j^n}{\tau} + V_j^n \frac{G_j^n - G_{j-1}^n}{h}\right] - \frac{a}{2}\left[\frac{T_{j-1}^n - 2T_j^n + T_{j+1}^n}{h^2} + \frac{T_{j-1}^{n+1} - 2T_j^{n+1} + T_{j+1}^{n+1}}{h^2}\right] = \frac{V_j^n \alpha}{mc_f}(T_k^{n+1} - T_j^n) \tag{6}$$

$$\frac{G_j^{n+1} - G_j^n}{\tau} = -\frac{V_j^n}{2m}\left[\frac{G_j^n - G_{j-1}^n}{h} + \frac{G_j^{n+1} - G_{j-1}^{n+1}}{h}\right], \tag{7}$$

T_k^{n+1} is expressed through the second equation of (4), G_j^{n+1} is found from equation (7) and T_j^{n+1} from equation (6).

This scheme yields a stable solution, i.e. moving temperature wave. With time, the temperature front curve becomes more slanting, which is due to heat transfer from air to a solid substance which results in a gradual heating of the solid phase. Additional modifications of the difference scheme (6) led to having stable non-oscillating solutions.

A certain decrease of pseudo viscosity coefficient can be reached by introducing a mass operator for time derivative [19]:

$$M_x \frac{\partial T}{\partial t} = \delta \frac{T_{j-1}^{n+1} - T_{j-1}^n}{\tau} + (1-2\delta)\frac{T_j^{n+1} - T_j^n}{\tau} + \delta \frac{T_{j+1}^{n+1} - T_{j+1}^n}{\tau}$$

$$M_x = \{\delta, (1-2\delta), \delta\} \quad \delta \leqslant 0.25.$$

Here $\delta > 0.1$ leads to the rise of the temperature profile up to and after the front line.

The effect of introduced pseudo viscosity can be diminished by means of the terms it pays of anti-diffusion flows [20]:

$$\frac{1}{2}\left[\beta_{j+1/2}\frac{T_{j+1}^n - T_j^n}{h} - \beta_{j-1/2}\frac{T_j^n - T_{j-1}^n}{h}\right].$$

The coefficients $\beta_{j\pm 1/2}$ are non-negative, confine anti-diffusion and are chosen in the following way:

$$\beta(R) = \begin{cases} 0, & R \leqslant \theta \\ \dfrac{(a+b(1-\vartheta))R}{(a+b)(1-\vartheta)} & \theta < R < 1-\vartheta \\ \dfrac{a+bR}{a+b} & |R-1| \leqslant \vartheta \\ \dfrac{(a+b(1-\vartheta))R - 2ab}{(a+b)(1-\vartheta)} & 1+\vartheta < R < 2 \\ \leqslant 2 & R \geqslant 2 \end{cases}$$

where

$$R_{j+1/Z} = \frac{T_j - T_{j-1}}{T_{j+1} - T_j} \quad R_{j-1/2} = \frac{T_{j+1} - T_j}{T_j - T_{j-1}},$$

ϑ, a, b are the constants, $0 < \vartheta < 1$, $a+b \neq 0$.

The effect of diffusion term can be decreased by varying the three additional parameters: a, b, θ.

However, the fundamental improvement mentioned in the above modifications of the difference scheme (6) is not given. This allows one to assume that in formulating the problem of equations (4) and (5), the first equation is sufficiently fully represented by means of scheme (6) realized with a Thomas algorithm [19].

ANALYSIS OF RESULTS

To compare the results obtained by using the system of equations (4) and (5) with the solution of the problem for the analogous case in ref. [1], the following initial data were applied [1]:

$$T_0 = 20°C \quad T_1 = 550°C \quad v_0 = 2.2\ \mathrm{m\,s^{-1}}$$

$$\lambda_k = 1.7\ \mathrm{W\,m^{-1}\,grad.} \quad m = 0.5$$

$$\rho_k = 2894\ \mathrm{kg\,m^{-3}} \quad c_k = 795\ \mathrm{J\,kg^{-1}\,grad.}$$

$$\rho_f = 348.3/\mathrm{T\ kg\,m^{-3}} \quad c_f = 1040\ \mathrm{J\,kg^{-1}\,grad.}$$

A series of calculations were performed at various initial parameters. The numerical value of the pseudo viscosity coefficient ranges from 0.45 to 0.55 at the given initial data. The analogous range of values is in other sources [19, 20] where the estimates, correcting the value of introduced pseudo viscosity are reduced. When conducting a numerical experiment, it may be concluded that the dumping of the oscillating solution occurs at these values. At $a > 0.55$, the width of the temperature front smearing increases.

Figure 1 shows the motion of temperature front along the tube at $a = 0.55$, with curves demonstrating temperature distribution every 2 h. As seen from Fig. 1. the temperature mark $T = 300°C$ moves with a constant velocity. In 6 h after the heated air is blown in, the exit temperature starts gradually growing, until reaching the entrance temperature.

The next two figures characterize some particular or numerical part of calculation, in particular, the effect of pseudo viscosity.

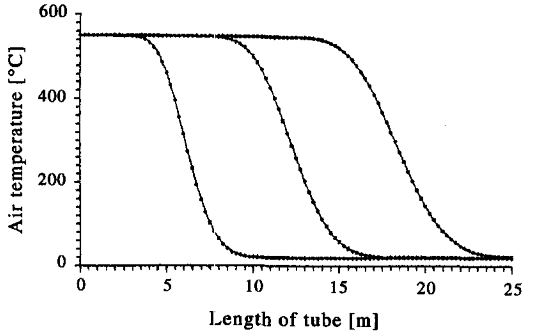

Fig. 1. Transient distribution of rock temperature in 2 h, basic calculation $a = 0.55$.

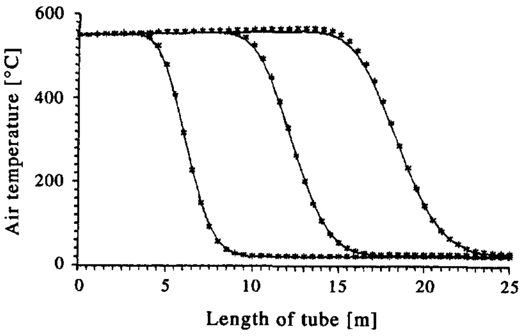

Fig. 4. Transient distribution of rock temperature for system with variable coefficients and with constant coefficients (asterisk).

As seen from Fig. 2, at a decrease of a ($a = 0.25$), the width of temperature front smearing decreases, but the solution becomes unstable.

Figure 3 shows the effect of the value of the pseudo viscosity coefficient in a wider range from $a = 0.055$ (curve 1—oscillating solution) to $a = 1.1$ (curve 3—the temperature front smearing), due to the increase of the effect of diffusion term, curve 2—at $a = 0.55$—basic calculation.

Figure 4 gives the results of calculation at $a = 0.45$ of system (4) and system of equation with the constant coefficient, i.e. with substitution of the first equation from equation (4) by equation (4″). This curve is marked by an asterisk. The results show that for the present problem the system of equations with constant coefficient can be used when conducting a calculation of specific applied problems, which essentially reduces computing time.

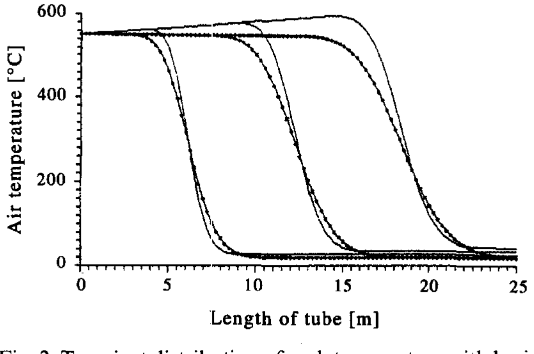

Fig. 2. Transient distribution of rock temperature with basic calculation (curve with asterisk) and with varied pseudo viscosity coefficient—$a = 0.25$.

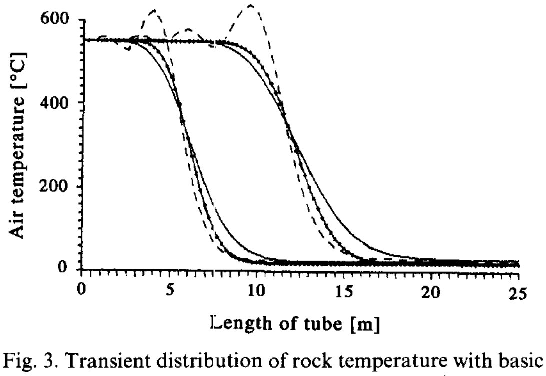

Fig. 3. Transient distribution of rock temperature with basic calculation (curve with asterisk) and with varied pseudo viscosity coefficient—$a = 0.055$ and $a = 1.1$ (dotted line).

CONCLUSION

(1) For the formulation of a problem for the heating of a solar thermal station reactor, a porous media model as a system of interacting imaginary media is used.

(2) The intercomponent heat transfer coefficient is important in the formation of the convective heat front and in the choosing of a way for its determination.

(3) For the obtained difference problem with the solution, which has a large gradient in restricted areas, the differential approximation method is used, which allows one to introduce an artificial viscosity coefficient and estimate this influence on the behavior of a difference solution. The proposed range of values of pseudo viscosity coefficient leads to the dumping of an oscillating solution and for the decreasing of the width of the temperature front smearing.

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